

CPT violations in Astrophysics and Cosmology

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Abstract

In this paper it is given a brief review of the current limits on the magnitude of CPT and Lorentz Invariance violations, currently predicted in connection with quantum gravity and string/M-theory, that can be derived from astrophysical and cosmological data. Even if not completely unambiguous, these observational tests of fundamental physics are complementary to the ones obtained by accelerator experiments and by ground or space based direct experiments, because potentially can access very high energies and large distances.

1 Introduction

Einstein (1905) introduced the postulate of the constancy of the velocity of light in empty space, justifying it on the bases of the negative result of the Michelson-Morley experiment (Michelson & Morley, 1887). Since then the covariance of physical laws under the group of Lorentz transformations, usually nicknamed the "Lorentz invariance" (LI), has been proven to hold locally within a very high accuracy in tests done on the Earth or nearby space. A modern version of the Michelson-Morley experiment has been performed comparing the frequency of solid state resonant cavities, in two orthogonal mode while in rotation respect to the Cosmic Microwave Background Radiation (CMBR) reference frame. Nevertheless the accuracy of the measurements is of the order of 10^{-7} for resonators in motion with the Earth (Saathoff et al., 2003) and of 10^{-10} for rapidly spinning ones (Stanwix et al., 2006). As we will discuss briefly in the remaining part of this introduction, this level of accuracy cannot exclude that Special Relativity Theory could be only an approximation of the physical reality, that could be violated by tiny perturbations introduced by conceivable mechanism.

Around the middle of the last century, the development of a fully Relativistic Quantum Field Theory (RFQT) has pointed out that there is a strong logical connection between the Lorentz invariance and the matter and antimatter duality. This is not unexpected since the very existence of antimatter is a

consequence of the relativistically covariant Dirac equation (Dirac, 1933). Following an early indication by Schwinger (1951), Pauli (1955) introduced the idea that for every process occurring in nature there is an allowed one, occurring with the same probability, in which each particle is replaced with its corresponding antiparticle, having reversed spin and following a trajectory that is the reflection under space and time inversion of the original ones. This is now called the “Schwinger-Pauli-Lüders CPT theorem” because Lüders (1957) showed that assuming

1. Lorentz Invariance;
2. Locality;
3. Unitarity;
4. Correct connection between spin and statistics

it is possible to prove that the successive application of charge conjugation C, parity reflection P and time reversal T in any order leave the Hamiltonian invariant. As a complement Greenberg (2002) has also proved the “inverse-CPT theorem”, showing that if under any circumstances CPT invariance is violated, then also Lorentz invariance will be.

In certain sense the CPT invariance is more fundamental than Lorentz invariance, because, as we will show later, it is possible to conceive modifications of the Hamiltonian of the fundamental interactions which violates Special Relativity, but are invariant under CPT transformations, acting equally on particles and antiparticles.

The CPT theorem points out what could be the physical origin of Lorentz violations. In particular the second hypothesis that leads to it is “locality”, which demands that space-like separated events should not influence each other (Wess, 1989). There are at least three good theoretical reasons to suspect that locality could not hold for arbitrary small distances:

1. The unavoidable singularities of the General Theory of Relativity (GRT) (Hawking, 1982) that makes the structure of space-time very complex at distances of the order of the Planck length $\ell_P = \sqrt{\hbar G/c^3} \simeq 1.66 \times 10^{-35}$ m. The fabric of space-time has been vividly described as a “foamy” structure (see *e.g.* Wheeler & Ford 1998), turbulently perturbed by black holes continuously popping out of the vacuum and evaporating in times of the order of $\Delta t = c\ell_P$.
2. String/M-theories are intrinsically non local theories (Amati et al., 1989) where the ordinary concept of point-like components of matter is substituted by two dimensional object with a finite, even if very small, dimensions. The space-time structure of the string theory is discontinuous on a scale $\Delta t \Delta x \geq c\ell_s^2$, being ℓ_s the size of the string (Yoneya, 1989). If string theory has incorporates gravity, one of the characteristic length of the

theory should be $\ell_s^{grav} = \ell_P$, but might exist other characteristic lengths, corresponding to different type of interactions, with $\ell_s > \ell_P$ (Lykken, 1996).

3. In string theory, gravity is just one of the many possible excitations of a string (or other extended object) living over some background metric space. The existence of such background metric space, over which the theory is defined, is needed for the formulation and for the interpretation of the theory. The “Loop Quantum Gravity” theory (Rovelli, 1998) is an attempt to eliminate this background space-time. In this theory the space-time has a kind of “polymeric” structure with minimal space cell with volumes $\Delta V \sim \ell_p^3$ (Rovelli & Smolin, 1995).

In any case the indeterminacy principle states that to resolve a length scale ℓ we need energies of the order of $\Lambda = \hbar c/\ell = (\ell_P/\ell)M_P$ where $M_P = \sqrt{\hbar c/G} = 1.22 \times 10^{19}$ GeV is the Planck mass. Naïve expectations of the orders of magnitude for *CPT* violations, motivated by dimensional considerations, will be

$$\Delta\mathcal{H} \approx \frac{E^2}{\Lambda} \quad (1)$$

At low energies the Hamiltonian is of the order of magnitude of the mass of the particle, therefore *CPT* symmetry can be tested in the laboratory measuring the difference of mass between particles and antiparticles. The best constraint has been obtained from the limits on the mass difference between the neutral strange mesons $K^0 - \bar{K}^0$ recently obtained from the KLOE experiment at the Daphne φ -factory (LNF-INFN)(Ambrosino et al., 2006)

$$\frac{|m_{K^0} - m_{\bar{K}^0}|}{m_{K^0}} < 1.26 \times 10^{-18} \quad (95\% \text{ C.L.}) \quad (2)$$

that from Eq. (1) is predicted to be $\approx 0.5 \times 10^{-19}$.

Astrophysical and cosmological test of the *CPT*/*LI* violations, that I will discuss in the rest of this paper are complementary to local tests, because test modifications of physical law over large spatial scale $D \sim 10^{26}$ m and long times $t_0 \sim 13.7$ Gy.

2 Parameterizations of the *CPT*/*LI* violations

Many different theoretical frameworks for *CPT*/*LI* have been investigated in detail (see e.g. Mattingly 2005 for a recent review). The closer to physical intuition is the “Modified Dispersion Relations” (MDR) framework, that has the advantage of supplying a relatively model independent parametrization of the *CPT*/*LI*, at the expense of rigor and completeness. Nevertheless this approach is, in my opinion for an experimentally oriented paper.

Assuming that the *CPT*/*LI* violating Hamiltonian of a free particle or field can be written $\mathcal{H} = \mathcal{H}_{free} + \Delta\mathcal{H}$ where $\Delta\mathcal{H}$ is a small perturbation of the

standard Hamiltonian $\mathcal{H}_{free} = \sqrt{p^2 + m^2}$, whose order of magnitude will be given by Eq. (1), we can put

$$E^2 - p^2 - m^2 = F(E, \vec{p}) \quad (3)$$

where the R.H.S. term of this equation can be expanded in Taylor series as

$$F(E, \vec{p}) = F_{\mu}^{(1)} p^{\mu} + F_{\nu\rho}^{(2)} p^{\nu} p^{\rho} + F_{\sigma\kappa\lambda}^{(3)} p^{\sigma} p^{\kappa} p^{\lambda} + \dots \quad (4)$$

where $p^{\mu} \equiv \{E, \vec{p}\}$ is the four momentum. It is worth noticing that we can derive many qualitative feature of CPT/LI violations from this relatively simple *Ansatz*.

- From a phenomenological point of view there is no *a priori* reason to expect that the coefficients in Eq. (4) are universal, even if the fundamental Lorentz violation is universal (for a discussion on this assumption see *e.g.* Alfaro 2005). At least we expect from Eq. (1) a dependance from the energy, that at low momentum is a dependence from the mass. In general we must assume an implicit dependence from all the conserved quantum numbers of the particle, namely intrinsic spin, charges, flavor, etc.
- Only the odd terms of Eq. (4) are CPT violating (“CPT odd”) while even terms are CPT conserving (“CPT even”), therefore we have the relation:

$$\overline{F}_{\mu_1\mu_2\cdots\mu_n}^{(n)} = (-1)^n F_{\mu_1\mu_2\cdots\mu_n}^{(n)} \quad (5)$$

where obviously $\overline{F}^{(n)}$ is the coefficient for the corresponding free antiparticle.

- Moreover the odd terms violate also P and T conjugation. This make a distinction between the right-handed and the left-handed component of a particle (or field) with spin, because under P and T the four-momentum of the field changes direction while the spin conserve its direction. In practice we will have

$$F_{\mu_1\mu_2\cdots k_{\mu}}^{(n)} = \zeta^n |F_{\mu_1\mu_2\cdots\mu_n}^{(n)}| \quad (6)$$

where $\zeta = \pm 1$ will be the polarization index of the particle, with $\zeta = +1$ if the spin is $\vec{s} \uparrow \uparrow \vec{p}$ and $\zeta = -1$ if $\vec{s} \uparrow \downarrow \vec{p}$. This fact induces a spin precession in the propagation of the particle in vacuum (birefrangence of the vacuum).

- From dimensional argument we expect

$$|F_k^{(1)} p^k| \approx |F_{ji}^{(2)} p^j p^i| \approx |F_{qrs}^{(3)} p^q p^r p^s| \sim \mathcal{O}\left(\frac{|p|^3}{\Lambda}\right) \quad (7)$$

while for $n > 3$ will be

$$\left| F_{\mu_1 \mu_2 \dots \mu_n}^{(n)} p^{\mu_1} p^{\mu_2} \dots p^{\mu_n} \right| \sim \mathcal{O} \left(\frac{|p|^n}{\Lambda^{n-2}} \right) \quad (8)$$

As a consequence at leading order it will be necessary to consider the firsts 3 terms of the dispersion relation's expansion, that are expected to have about the same order of magnitude.

3 Tests on a preferential direction in space-time

The coefficient of the first term on R.H.S. of Eq. (4) is a four-vector with the dimensions of a mass, that assign a preferential direction in space-time, sometimes called in the literature the ‘‘Chern-Simons term’’. We can write the MDR of a photon, including only the directional term in the form

$$\omega = \sqrt{k^2 + 2\zeta_\gamma(\xi_0\omega - \boldsymbol{\xi} \cdot \vec{k})} \quad (9)$$

where we have put $F_\mu^{(1)} \equiv \{\xi_0/2, -\boldsymbol{\xi}\}$. Solving this equation we have the explicit form of the MDR

$$\omega = \zeta_\gamma \xi_0 \pm \sqrt{k^2 + \zeta_\gamma^2 \xi_0^2 - 2\zeta_\gamma \boldsymbol{\xi} \cdot \vec{k}} \simeq \pm k + \zeta_\gamma(\xi_0 \mp |\boldsymbol{\xi}| \cos \theta)$$

where θ is obviously the angle between \vec{k} and $\boldsymbol{\xi}$. The ambiguity of the sign is only apparent because the lower sign is meaningful only in the case that $k < 0$, but in this case $\cos \theta < 0$. Therefore the true physical solution is

$$\omega \simeq k + \zeta_\gamma(\xi_0 - |\boldsymbol{\xi}| \cos \theta) \quad (10)$$

As we said before the polarization index ζ_γ is +1 for a right-handed circularly polarized wave and -1 for the opposite polarization, therefore this vacuum birefringence effect could be detected in the propagation of polarized radiation. A linearly polarized wave is represented by the superposition of two circularly polarized waves

$$\Psi = \Psi_0 \{ e^{-i\alpha - \omega_+ t} \boldsymbol{\epsilon}_+ + e^{i\alpha - \omega_- t} \boldsymbol{\epsilon}_- \} \quad (11)$$

where α is the initial polarization angle. It is evident that the polarization angle as a function of time will be $\alpha(t) = \alpha_0 + (\omega_+ - \omega_-)t$ from which we have that the polarization plane of radiation emitted by a source at cosmological redshift z will rotate at Earth by an angle

$$\Delta\alpha = 2 \int_0^z (\xi_0(z) - |\boldsymbol{\xi}(z)| \cos \theta) \frac{dz}{(1+z)H(z)} \quad (12)$$

It is interesting that this rotation would be independent upon the wavelength of the radiation, being distinguishable from the interstellar Faraday rotation that is $\propto \lambda^2$.

Nodland & Ralston (1997) claimed that data on polarized radiation emitted by distant radio galaxies show a marginal statistical evidence (3σ) for a systematic rotation depending on the angle θ between the propagation wave vector \vec{k} of the radiation and a direction roughly localized in a region $19h \leq \alpha \leq 23h$, $-20^\circ \leq \delta \leq +20^\circ$, that could be explained by $|\xi| \simeq 10^{-41}$ GeV. But this claim was not confirmed by a reanalysis of the same data with different statistical techniques (Loredo et al., 1997). A search to a sample of 160 radiogalaxies with $0.3 < z < 2.12$ Carroll & Field (1997) found

$$\xi_0 = (0.8 \pm 1) \times 10^{-41} h_0 \text{ GeV} \quad |\xi| = (1.5 \pm 1.9) \times 10^{-41} h_0 \text{ GeV} \quad (13)$$

Independently Wardle et al. (1997) from observation of the polarization in the optical V band of 3C265 ($z = 0.82$) found a mean deviation of $-1.4^\circ \pm 1.1^\circ$, that yields a limit from Eq. (12) at present time, assuming a moderate evolution $|\xi| \propto (1+z)$ the upper limit

$$|\xi| \leq 2 \times 10^{-41} \text{ GeV} \quad (95\% \text{ C.L.}) \quad (14)$$

This limit indicates that a directional term is strongly suppressed, even respect to the dimensional estimate $(\hbar\omega)^2/M_P \sim 4 \times 10^{-37} \text{ GeV}$.

Feng et al. (2006) speculate that a preferred space-time direction in the Universe could be originate by a scalar field ϕ that might constitute the so called “quintessential dark energy” (for a recent review see e.g. Copeland et al. 2006). In this case in the CMBR rest frame the four-vector ξ_μ is time-like, being $\xi_0 = \dot{\phi}$. We can estimate the order of magnitude of the vector from observations of the expansion rate of the Universe using the equation (Ratra & Peebles, 1988)

$$\dot{\phi}^2 = \frac{16\pi}{M_P^2} (1 + w_\phi) \rho_\phi \quad (15)$$

where ρ_ϕ is the dark energy density and $p_\phi = w_\phi \rho_\phi$ its equation of state. A recent estimate (Riess et al., 2004) sets an upper limit $w \leq -0.76$ at 95% C.L. from which, assuming $\Omega_V = 0.732 \pm 0.018$ (Spergel et al., 2006), we estimate from Eq. (15) $\xi'_0 < 10^{-41} h_0 \text{ GeV}$. In the solar system frame, moving with $V \simeq 370 \text{ km/s}$ respect to CMBR, the components of the four vector would be $\xi_0 = \gamma \xi'_0$ and $|\xi| = \gamma \beta \xi'_0 \simeq 1.2 \times 10^{-3} \xi_0$.

4 Test of CPT-odd violations from the polarization of the CMBR

In the MDR framework, expressed by the expansion of Eq. (4) in a space-time isotropic Universe, neglecting directional effects that appears from experiments very suppressed, the dispersion relation for photons at leading order is

$$\omega = \sqrt{k^2 + \zeta_\gamma \frac{\eta_\gamma}{M_P}} k^3 \simeq k \left(1 + \zeta_\gamma \frac{\eta_\gamma}{M_P} \frac{k}{2} \right) \quad (16)$$

where η_γ is an adimensional parametrization of the magnitude of CPT-odd LI violations. From this dispersion relation, we obtain the phase velocity of light

$$c_\gamma(\omega, \zeta_\gamma) = \frac{\omega}{k} \simeq 1 + \zeta_\gamma \frac{\eta_\gamma}{M_P} \frac{\omega}{2} \quad (17)$$

where we have preserved the measured value $c_\gamma = 1$ for $\omega \ll M_P$.

The two circular polarization of the photons will propagate with different phase velocity (cosmological birefringence)

$$\Delta v(\omega) = c_\gamma(\omega, +1) - c_\gamma(\omega, -1) = \frac{\eta_\gamma}{M_P} \omega \quad (18)$$

As in the case of directional term, illustrated in the previous §3, the plane of polarization of a linearly polarized wave from a source at redshift z is rotated of an angle, that in this case, substituting

$$\omega(z) = \frac{2\pi c_\gamma}{\lambda} (1+z) \quad (19)$$

where λ will be the detected wavelength of the photon, will be

$$\Delta\alpha(z) \simeq \frac{2\pi}{M_P} \lambda^{-1} \int_0^z \frac{\eta_\gamma(z)}{H(z)} dz \quad (20)$$

We observe that the adimensional CPT-odd coefficient is expected to be by dimensional argument $\eta_\gamma \propto \hbar\omega/\Lambda$, making the effective dependence of the rotation angle $\propto \lambda^{-2}$, distinguishable from the interstellar Faraday rotation that is as we said before $\Delta\alpha_F \propto \lambda^2$.

The detailed maps of CMBR temperature and polarization, obtained from WMAP (Page et al., 2006), offer an intriguing possibility to set limits to the cosmological birefringence at redshifts $0 \leq z \leq 1100$ (as proposed earlier by Lue et al. 1999).

- From fits of WMAP and BOOMERANG data Feng et al. (2006); Xia et al. (2007) obtain the limit

$$\Delta\alpha = -6.2^\circ \pm 3.8^\circ$$

- From wavelet fits of WMAP 3-year data Cabella et al. (2007) obtain a rotation of polarization of the CMBR

$$\Delta\alpha = -2.5^\circ \pm 3^\circ$$

The rotation of polarization expected from Eq. (20), averaged over the spectrum of the CMBR, assuming an evolution $\eta_\gamma(z) = \eta_\gamma(0)(1+z)$, is

$$\Delta\alpha_{CPT} \simeq 11.7^\circ h_0^{-1} \eta_\gamma \quad (21)$$

The result of Cabella et al. (2007) is close to being a significant negative experiment, because gives the upper limit

$$\eta_\gamma < 0.2 h_0 \quad (95\% \text{ C.L.}) \quad (22)$$

that imposes the energy scale of CPT-odd violations for photons to be $\Lambda \geq 5 h_0^{-1} M_P$.

5 Test of CPT-odd violations from GRB's

Unpolarized radiation can be represented by the superposition of two equal amplitude waves, with opposite circular polarization. The group velocity of the photon in vacuum will at leading order will be

$$v_\gamma(\omega, \zeta_\gamma) = \frac{\partial \omega}{\partial k} \simeq 1 + \zeta_\gamma \frac{\eta_\gamma}{M_P} \omega \quad (23)$$

slightly different from the phase velocity given by Eq. (17). This introduce a time spread $\Delta t \propto v_\gamma(\omega, +1) - v_\gamma(\omega, -1)$ that is, for a source at redshift z given by

$$\Delta t(z) \simeq \frac{4\pi}{M_P} \lambda^{-1} \int_0^z \frac{\eta_\gamma(z)}{H(z)} dz \quad (24)$$

where we will assume as in the previous section $\eta_\gamma(z) = \eta_\gamma(0)(1+z)$.

Amelino-Camelia et al. (1998a) suggested that GRB could be used to constraint the vacuum dispersion of radiation, due to their short intrinsic duration and high energy emission. However it appears that the present limits that can be obtained by this method cannot really access the Planck scale. In fact from Eq. (24) we derive a time spread for a burst in the hard X-ray band

$$\Delta t(z=1) \simeq 22.7 \left(\frac{\hbar \omega}{200 \text{ keV}} \right) \eta_\gamma h_0^{-1} \mu s \quad (25)$$

The shortest time scale ever detected has been observed in the exceptional GRB920229 (Schaefer & Walker, 1999), observed by BATSE to have a rise time $\tau = 220 \pm 30 \mu s$. From the negligible time dispersion of the rise of the burst among the low energy channel (25-50 keV) and the most populated channel (100-300 keV), that we estimate $< 130 \mu s$, and assuming a redshift $z \approx 1$ (Amelino-Camelia et al., 1998b), we can set a limit $\eta_\gamma < 5 - 6 h_0$ which implies $\Lambda > 0.2 h_0^{-1} M_P$. Similar limits are obtained with an accurate statistical analysis of a sample of 35 GRB's with known redshift (Ellis et al., 2006).

The observation of linear polarization of the prompt emission from GRB at cosmological distances could set stringent limits to the birifrangent propagation, that is implied by Eq. (17). In fact linear polarized γ -ray photon with different energies will be rotated by an amount given by Eq. (20). Therefore the observation of linear polarizations in the prompt emission of GRB's could give a very strong limit to the vacuum birifrangence (Mitrofanov, 2003).

Fan et al. 2006 from the observation of the afterglows of GRB020813 and GRB 021004 in the UV band could set the limit $|\eta_\gamma| \leq 10^{-7}$, if one can rule out an intrinsic origin for the rotation of the polarization vector at various energies.

6 Tests on CPT violations from the Crab Nebula

The dispersion relation of charged particles (electron or protons), in a space-time isotropic Universe, can be put in the form (Myers & Pospelov, 2003)

$$E(p, \zeta_p) = \sqrt{m^2 + (1 + \epsilon_p)p^2 + (\eta'_p + \zeta_p \eta_p) \frac{p^3}{M_P}} \quad (26)$$

where the coefficients ϵ_p , η_p and η'_p are adimensional, expected to be $\propto M_P/\Lambda$. In this formula we have introduced a distinction between the C-even part of the coefficient of cubic modification to the dispersion relation η'_p and its C-odd part η_p . It is evident from this expression that the maximum attainable phase velocity of particle, for $E \ll M_P$, will be

$$c_p \simeq \sqrt{1 + \epsilon_p} \neq 1 \quad (27)$$

In the moderate ultrarelativistic regime $m \ll E \ll M_P$ we assume the C-even parameter $\eta'_p = 0$ and we have

$$E \simeq pc_p \left(1 + \frac{m^2}{2p^2 c_p^2} + \zeta_p \frac{\eta_p}{2M_P c_p^2} p \right) \quad (28)$$

Deriving this equation and substituting $E \approx pc_p$ we find the group velocity of the De Broglie wave associated to the particle

$$v(E, \zeta_p) = \frac{\partial E}{\partial p} \simeq 1 - \frac{m^2}{2E^2} + \zeta_p \frac{\eta_p}{M_P c_p^2} E \quad (29)$$

From this we calculate the energy and helicity dependent Lorentz factor at leading order

$$\gamma(E, \zeta_p) = \frac{1}{\sqrt{1 - \frac{v^2}{c_p^2}}} \simeq \left(\frac{m^2}{E^2} - 2\zeta_p \frac{\eta_p}{M_P c_p^2} E \right)^{-1/2} \quad (30)$$

The peculiarity of this formula is that for $\zeta_p = +1$ (right-handed particles) the Lorentz factor shows an apparent divergence that is likely canceled by higher order terms, while for $\zeta_p = -1$ (left-handed particles) it has a maximum value $\gamma_{max} \simeq 1.7 \times 10^7 / \eta_p^{1/3}$ for $E \simeq 14.7 / \eta_p^{1/3}$ TeV.

Several observable modifications of familiar e.m. radiation processes follow from this fact, as we will show in the rest of this section, can be understood easily from kinematical considerations. We begin with the Compton scattering

$$e^\pm + \gamma \rightarrow e^\pm + \gamma \quad (31)$$

Following a well known method (Blumenthal & Gould, 1970) we consider the scattering as occurring in the Thomson limit $\tilde{\omega}' \simeq \tilde{\omega}$ in the rest frame of the electron, where $\tilde{\omega}$ and $\tilde{\omega}'$ are the incoming and outgoing energy of the photon. In order to calculate these energies in the electron rest frame, we consider that from Eq. (16) follows that, also in presence of CPT/LI violations we have

$$\omega^2 - k^2 c_\gamma^2(\omega, \zeta_\gamma) = 0 \quad (32)$$

in any inertial reference. Condition that is realized by the pseudo-Lorentz transformations:

$$\omega' = \omega \frac{c_\gamma - v \cos \theta}{\sqrt{c_\gamma^2 - v^2}} \quad ; \quad k' = \frac{\omega'}{c'_\gamma} \quad ; \quad \tan \theta' = \frac{\sqrt{c_\gamma^2 - v^2} \sin \theta}{c_\gamma \cos \theta - v} \quad (33)$$

Using these transformations we have the energy of the incoming and outgoing photons in the electron rest frame:

$$\tilde{\omega} = \omega \frac{c_\gamma(\omega, \zeta_\gamma) - v \cos \theta}{\sqrt{c_\gamma^2(\omega, \zeta_\gamma) - v^2}} \quad ; \quad \omega' = \tilde{\omega}' \frac{c_\gamma(\tilde{\omega}', \zeta_\gamma) - v \cos \theta'}{\sqrt{c_\gamma^2(\tilde{\omega}', \zeta_\gamma) - v^2}} \quad (34)$$

The maximum energy in the laboratory of the scattered photon will be approximately

$$\omega_{IC}^{max} \simeq \omega \frac{c_\gamma(\omega') + v}{c_\gamma(\omega) - v} \quad (35)$$

that tends to the well known standard expression $\omega_{IC}^{max} = \omega \gamma^2 (1 + \beta)^2$ in the Lorentz invariant limit.

We observe that the corrections to the nominator of Eq. (35) are negligible, while they determine the order of magnitude of the denominator. In practice we will put $c_e = 1$ (see later in this section for a justification of this assumption) and use the approximation

$$\omega_{IC}^{max} \simeq 4\omega \left(\frac{m^2}{E^2} + \zeta_\gamma \frac{\eta_\gamma}{M_p} \omega - 2\zeta_e \frac{\eta_e}{M_p} E \right)^{-1} \quad (36)$$

Viewing the magnetic field as a collection of virtual photons with average energy $\omega_B = eB/m$ that are scattered by the fast electrons (see *e.g.* Lieu & Axford 1993), we can apply the theory of inverse Compton scattering in presence of Lorentz violations outlined above. Therefore the maximum energy of the synchrotron photons will be given by Eq. (36) in the form

$$\omega_{Sinc}^{max} \simeq 4\omega_B \left(\frac{m^2}{E^2} + \zeta_\gamma \frac{\eta_\gamma}{M_p} \omega_B - 2\zeta_e \frac{\eta_e}{M_p} E \right)^{-1} \quad (37)$$

In the general case $\omega_B \ll E$ we can neglect the effect of CPT-odd violations of photon propagation, that is constrained from Eq. (22) to be $\eta_\gamma \leq 0.14$ (assuming $h_0 = 0.7$). Neglecting a quantity $\leq 2 \times 10^{-33}$ the term in parenthesis at the

R.H.S. of Eq. (37) is exactly equal to the Lorentz factor of the electron given by Eq. (30).

As we have noted above this formula has a peculiar behavior for left-handed particle, but in usual conditions the electrons are a mixture of left-handed and right-handed components. However for massless particle the helicity is a good quantum number, therefore when $E \gg m$ electrons(positrons) are almost all left-handed(right-handed) like neutrinos(antineutrinos). Following Jacobson et al. (2003) we maximize the equation

$$\omega_{Sync}^{max} = \frac{eB}{E} \left(\frac{m^2}{E^2} + 2 \frac{\eta_{e^-}}{M_P} E \right)^{-\frac{3}{2}} \quad (38)$$

taking E as an independent variable. The maximum is attained for an energy of the electron $E = 10/\eta^{1/3}$ TeV with a Lorentz factor $\gamma(E) = 1.58 \times 10^7/\eta^{1/3}$, that gives the constraint

$$\eta_{e^-} \leq \frac{M_P}{m} \left(\frac{0.35eB}{m \omega_{Sync}^{max}} \right)^{\frac{3}{2}} \quad (39)$$

The Crab Nebula is an excellent astrophysical laboratory for testing this type of CPT-odd violations. Assuming that synchrotron emission contribute to the γ -ray unpulsed spectrum of the Crab Nebula up to a maximum energy of ≈ 100 MeV Jacobson et al. (2003) derive a constraint to the CPT-odd violation parameter

$$\eta_{e^-} \leq 7 \times 10^{-8} \left(\frac{\omega_{Sync}^{max}}{100 \text{ MeV}} \right)^{-3/2} \left(\frac{B}{0.6 \text{ mG}} \right)^{3/2} \quad (40)$$

It is worth noticing that this limit is rather conservative, because the EGRET spectrum of the unpulsed component of the Crab Nebula (Nolan et al., 1993) has a break at ~ 1 GeV, suggesting a limit ~ 30 times smaller. Nevertheless we must point out that the limit obtained is consistent only if would be possible to exclude that the synchrotron radiation is emitted by e^+e^- pairs, because in this case the limits applies only to the electron component.¹

No better limit can be obtained from the cutoff in the UHE part of the spectrum of the nebula, observed by the HESS experiment at 14.3 ± 2 TeV (Masterson & et al., 2005). The emission of the nebula above ~ 1 GeV is extremely well consistent (de Jager & Harding, 1992; Atoyan & Aharonian, 1996) with the expected flux produced by Compton scattering of synchrotron, dust IR, and/or CMBR photons by the same electrons that produce the synchrotron component. In fact if we apply Eq. (36) derived above, we would expect in any case $\omega_{IC}^{max} \gg 100$ TeV. Very likely the cause of the ~ 10 TeV cut-off is to be searched in the absorption process $\gamma + \gamma \rightarrow e^+ + e^-$ that is very effective when $\omega_{IC}\omega_{IR} \geq m^2$ (Telnov, 1990).

¹After the presentation of this paper an extended analysis of the effect of LV on the Crab Nebula emission has been discussed by Maccione et al. 2007, with interesting results.

In the derivation of the Eq. (36) we have neglected ϵ_e . It is worth noticing that from the UHE emission from The Crab Nebula we can set also a stringent limit to the CPT-even LI violations. We consider the process

$$\gamma \rightarrow e^+ + e^- \quad (41)$$

namely pair creation in vacuum. This process is forbidden by LI because the conservation of four-momentum imposes

$$\omega^2 - k^2 = (E_+^2 - p_+^2) + 2(E_+E_- - p_+p_- \cos \theta) + (E_-^2 - p_-^2) \quad (42)$$

where the L.H.S. is null and the R.H.S. is $> 2m^2$. If we use the MDR of Eq. (26) including only the CPT-even term, Eq. (42) is written

$$\omega^2 - k^2 = 2m^2 + \epsilon_e(p_+^2 + p_-^2) + 2(E_+E_- - p_+p_- \cos \theta) \quad (43)$$

that can be satisfied if $\omega = k$ when $\epsilon_e < 0$. The threshold for the occurrence of the reaction (41) in presence of CPT-even/LI violations is

$$\omega \geq \frac{2m_e}{\sqrt{-\epsilon_e}} \quad (44)$$

The fact that photons with $E_\gamma > 20$ TeV have been observed from the Crab Nebula set the constraint

$$|c_e^2 - 1| < 2.5 \times 10^{-15} \quad (45)$$

comparable with the limit inferred from observations of VHE γ -rays from the extragalactic source MK501 (Stecker & Glashow, 2001).

7 Global tests of special relativity

As we have discussed before, the leading order CPT-even term in the expansion of Eq. (4) induces violation of Special Relativity that could be observed experimentally, because the limiting velocity for massive particle is changed by Eq. (27) (Coleman & Glashow, 1999).

An important QED process is the Čerenkov emission in vacuum by protons

$$p \rightarrow \gamma + p \quad (46)$$

This process is also forbidden in vacuum by LI because the four-momentum conservation imposes

$$E^2 - p^2 = E'^2 - p'^2 + 2(\omega E' - kp' \cos \theta) \quad (47)$$

but if LI holds the L.H.S of this equation is $E^2 - p^2 = m_p^2$ while the r.h.s. is always $> m_p^2$. On the contrary if LI is violated from Eq. (26) we have

$$\epsilon_p(p^2 - p'^2) = 2(\omega E' - kp' \cos \theta) \quad (48)$$

that has solutions for $\epsilon_p > 0$. Therefore the reaction (46) occurs over the threshold

$$p > \frac{2m_p}{\sqrt{3\epsilon}} \quad (49)$$

From the simple facts that HiRes (Abbasi et al., 2007) and AGASA (Takeda et al., 1998) has observed primary cosmic rays with energies up to $E_p \sim 10^{19}$ eV we can derive the constraint

$$c_p^2 - 1 < 1.3 \times 10^{-20} \quad (50)$$

Greisen (1966) and Zatsepin & Kuz'min (1966) published independently a calculation of the interaction of protons with the CMBR that should prevent cosmic rays with energy $\geq 10^{20}$ eV to cross distance $d \leq 50$ Mpc (the so called GZK cut-off). The experimental situation is not clear at the moment, because the data from the HiRes experiment (Abbasi et al., 2007) show an evidence at 5σ for the presence of a sharp cut-off at 6×10^{19} eV while the AGASA data (Takeda et al., 1998) show a flattening (ankle) of the spectrum above 10^{19} eV, reconstructing six events with energy $\geq 10^{20}$ eV.

Hopefully the experimental situation will be resolved in one way or the other by the Pierre Auger Observatory, expected to be fully operational in few months from now, because Glashow (1999) showed that UHE cosmic rays can play an important role in connection with tests of Special Relativity.

The reaction producing the GZK cut-off is the photoproduction reaction

$$p + \gamma \rightarrow \Delta^+ \quad (51)$$

where Δ^+ is the lowest pion-nucleon resonance with a mass $m_\Delta = 1232$ MeV. The four momentum conservation gives

$$E_p^2 - p_p^2 + 2(\omega E_p - k p_p \cos \theta) = E_\Delta^2 - p_\Delta^2 \quad (52)$$

Using the MDR expansion in the ultrarelativistic regime $E_p \gg m_p$ we have

$$m_p^2 + \epsilon_p p_p^2 + 2\omega p_p (\sqrt{1 + \epsilon_p} - \cos \theta) = m_\Delta^2 + \epsilon_\Delta p_\Delta^2 \quad (53)$$

In the LI case $\epsilon_p = \epsilon_\Delta = 0$ the threshold for this reaction is

$$p_p \geq \frac{m_\Delta^2 - m_p^2}{4\omega} \simeq 5 \times 10^{19} \text{ eV}/c \quad (54)$$

perfectly consistent with the observed cut-off. However we can approximate $p_\Delta \simeq p_p$ because $k \ll p_p$ therefore Eq. (53) can be written

$$(\epsilon_p - \epsilon_\Delta) p_p^2 + 2\omega p_p \leq m_\Delta^2 - m_p^2 \quad (55)$$

that can be satisfied only if

$$\epsilon_\Delta - \epsilon_p < \frac{\omega^2}{m_\Delta^2 - m_p^2} \simeq \times 10^{-25} \quad (56)$$

This shows that a tiny LI violation can eliminate the GZK cut-off. It is clear at this point that a secure confirmation of the observation of the GZK cut-off would set a very strong limit to the validity of Special Relativity. In fact from the dimensional estimate we expect $\epsilon_\Delta - \epsilon_p \sim m_\Delta - m_p/M_P \sim 2.4 \times 10^{-20}$, more then 4 order of magnitudes larger then the limit that could be inferred from the confirmation of the GZK cut-off.

8 Conclusions

Astrophysical tests show that Einstein's Special Relativity Theory is in quite good health, far beyond the Planck scale. The final assessment of the evidence for GZK cut-off in the primary UHE cosmic ray spectrum would secure an upper limit to CPT-even violations $\lesssim 10^{-25}$.

The Crab Nebula promises to be an excellent particle physics laboratory for the search of CPT-odd effects, that could allow the exploitation of electron beam energies up to ≈ 2500 TeV, but unfortunately the interpretation of data is far from being lacking in ambiguity. For example if it could be demonstrated that synchrotron emission is produced by negative electrons only, a limit to CPT-odd violations of the order of $\lesssim 10^{-8}\ell_P$ could be assessed.

The existence of a preferred direction in space-time, possibly connected with a quintessential dark-energy, is constrained by the optical polarimetry of far distant galaxies to be very small $< 5 \times 10^{-5}\ell_P$, but the scale of anisotropy estimated from the dark energy density is $< 2.5 \times 10^{-5}\ell_P$, close but not conclusive. It is intriguing that in the future X and Γ -ray polarimetry of bright objects at cosmological distances, like AGN and GRB, could improve the present limits, by order of magnitudes, if emission models are also improved.

The polarization of the CMBR is another interesting source of data on possible CPT-odd violations in the photon sector, but at present the results of WMAP allow to constraint the scale in the range $\sim 0.1\ell_P$. The predicted sensitivity of the Planck satellite, to be launched about one year from now, can improve significantly the above limit in the next decade.

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